

Deduction rules

The deduction rule in the logical calculus of association rules is a relation of the form

$$\frac{\alpha_1, \alpha_2, \dots, \alpha_n}{\beta}$$

where $\alpha_1, \alpha_2, \dots, \alpha_n, \beta$ are association rules. This deduction rule is correct if it holds for each data matrix M : If $\alpha_1, \alpha_2, \dots, \alpha_n$ are true in M , then also β is true in M .

We are interested in correct deduction rules of the form

$$\frac{\varphi \approx \psi}{\varphi' \approx \psi'}$$

where $\varphi \approx \psi$ and $\varphi' \approx \psi'$ are association rules.

Such deduction rules can be used namely in the following ways:

- *To reduce the output of a data mining procedure:* If the association rule $\varphi \approx \psi$ is a part of a data mining procedure output (i.e. it is true in an analysed data matrix) and if

$$\frac{\varphi \approx \psi}{\varphi' \approx \psi'}$$

is the correct deduction rule then it is not necessary to put the association rule $\varphi' \approx \psi'$ into the output. The used deduction rule must be transparent enough from the point of view of the user of the data mining procedure. An example of a simple deduction rule is a dereduction deduction rule

$$\frac{\varphi \Rightarrow^* \psi}{\varphi \Rightarrow^* \psi \vee \chi}$$

that is correct for each implicational quantifier \Rightarrow^* [Ha 78].

- *To decrease the number of actually tested association rules:* If the association rule $\varphi \approx \psi$ is true in the analysed data matrix and if

$$\frac{\varphi \approx \psi}{\varphi' \approx \psi'}$$

is the correct deduction rule, then it is not necessary to test $\varphi' \approx \psi'$.

Thus it is reasonable to ask when the deduction rule of the form

$$\frac{\varphi \approx \psi}{\varphi' \approx \psi'}$$

is correct. It can be shown that there are several propositional formulas Φ, Ψ derived from $\varphi, \psi, \varphi', \psi'$ such that this deduction rule is correct if and only if Φ and Ψ are tautologies of the propositional calculus [Ra 98A], [Ra 98C]. The propositional formulas Φ, Ψ depends on the class of 4ft-quantifiers the quantifier \approx belongs to.

Some of these deduction rules are applied in the procedure 4ft-Miner.